

# Overcoming Challenges in Hybrid Simulation Design and Experiment

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**Abstract.** The purpose of this paper is to present the concept of modules and interfaces for a hybrid simulation model that forecasts demand for healthcare services on the regional level. The interface, developed with the Visual Basic for Application programming tools for spreadsheets, enables comprehensive planning of simulation experiment for the combined model that operates based on two different simulation paradigms: continuous and discrete-event. This paper presents the capabilities of the developed tools and discusses the results of the conducted experiments. The cross-sectional age-gender specific demographic parameters describing population of two subregions of Lower Silesia were calculated based on historical data retrieved from Central Statistical Office databases. We demonstrated the validity of the developed interface. The model correctly responded to the seasonal increased intensity of patients arrivals to healthcare system.

**Keywords:** Healthcare services · Simulation modeling Continuous simulation · Discrete-event simulation · Hybrid simulation

# 1 Introduction

This paper builds up on our previous study that focuses on the use of combined simulation methods to support healthcare demand predictions [14, 15]. The hybrid model simulates the consequences of the demographic changes, the variability in the incidence rates that result from the population ageing, and the seasonal fluctuations in epidemic trends on the future demand for healthcare services. This in turn may help the healthcare managers to adjust the resources needed to cover the future healthcare needs expressed by the population inhabiting the region.

This is still the on-going project that aims to develop a fully operative hybrid model that combines two simulation approaches: continuous and discrete-event. Hitherto, we were able to solve the "drainage problem" in the aging chain demographic simulation [18] and we proposed a method to eliminate the differences between historical data and simulation results when projecting the population evolution within the predefined time range. This was achieved by designing the hierarchical blocks and increasing the number of elementary cohorts up to 210 elementary one-year male/female items. In our research we were faced however with another challenge. Simulation experiments have revealed that due to the very large number of results (millions of records) it was necessary to

J. Świątek et al. (Eds.): ISAT 2018, AISC 853, pp. 207-217, 2019.

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https://doi.org/10.1007/978-3-319-99996-8\_19

develop a set of analytical tools for simulation experiment planning. It was also essential to construct the appropriate data sheets for the fast and accurate input/output data analysis, especially when the more advanced sampling methods are applied.

The overall aim of this paper is to present the approach for credible experimental design and output data analysis to be applied in the hybrid simulation model.

# 2 Theoretical Background - Premises for Hybrid Solution

Literature survey proves that simulation is widely and successfully used in healthcare decision making [7, 10]. Simulation methods may be divided into different categories based on various criteria, whereas practice in the area of health care applications indicates that the most common criterion [2, 9, 12, 13, 16] is related to time perception. According to this criterion, the simulation methods are divided into:

- · Monte Carlo techniques which, generally, ignore the passage of time,
- continuous modeling that considers the cause-and-effect relationships, feedback loops, and fixed-interval time steps,
- discrete-event modeling that registers changes caused by individual objects moving through the system and random-interval time steps closely related to state changes occurring in the system,
- agent-based system, the sub-method of discrete modeling with the ability to focus on the behaviors and interactions between particular objects.

The type of the problem determines the simulation approach best fitted to model the issue. For example, when modeling the factors affecting the epidemic health condition [8] or the susceptibility to a given type of disease a continuous approach is preferred. However, to model a performance of a health care facility one usually selects discreteevent approach [7]. Factors that cannot be identified with certainty lead to stochastic simulation techniques [3], i.e. Monte Carlo or discrete-event.

When modelling the performance of health care systems the specific concepts appear: *cohort modelling* is useful to represent the flow of individuals between agegender groups; *temporal factors* such as hour of day, day of week, month, season, calendar year are helpful to describe the patients arrival rates to facilities; *geographic characteristics* such as the distance to the facility may be used to determine the reaction time needed to effectively provide emergency service. Each of these concepts is usually more closely connected with only one simulation approach. For example, the temporal changes are more easily managed using discrete-event simulation, while cohort modelling is more typical for continuous modelling.

Due to the heterogeneity of approaches used in the simulation of health care systems, hybrid concepts [17] have been developed to combine different methods in one master model [1, 4, 6]. In our study we applied three approaches: Monte Carlo in the context of repetitive experiments and sampling, continuous simulation to model demographic evolutions, and discrete-event method to generate objects representing patients arriving to a healthcare facility with service requests. One of the benefits of such a solution, observed also in our study, is the ability to consider large scale problems, i.e. many millions of patients arriving to health care facilities, [5].

### **3** Description of the Hybrid Model

#### 3.1 Basic Assumptions

The hybrid model consists of two submodels: continuous model created in accordance with the system dynamics approach and discrete model built in accordance with the discrete-event approach. The continuous model performs demographic simulations for the years 2010–2030 based on historical data for the period 2010–2015 and official governmental forecasts describing the expected population changes. The discrete model uses demographic data from the continuous model, empirical data on hospital admissions drawn from National Health Fund regional branch and the elaborated parameters that describe seasonality trends occurring in patients arrivals.

#### 3.2 Population Model

The first model (see Fig. 1) is essential for predicting population aspects: the population size, the number of births and deaths, migration and growing up processes. The model was built in Extendsim [11] and the detailed description of this model may be found in [18]. In order to increase the clarity of the text the brief recapitulation is given below.



Fig. 1. An excerpt from the first model: continuous simulation approach (young males part).

There are two gender chains, female (F) and male (M), and each chain consists of 18 major cohorts. All major cohorts, except the oldest, consist of 5 elementary cohorts Each chain has two special-type cohorts: marginal left ( $F_0_4$  and  $M_0_4$ ) and marginal right ( $F_85+$  and  $M_85+$ ). The youngest cohort (marginal left) interacts with a stream of births (inflow), while the oldest cohort (marginal right) contains a large number of (20) elementary cohorts representing the entire population of the oldest people. Each cohort also interacts with one or two streams such as a growing up stream, a stream of deaths (outflow) and a stream of migration (balance of inflow and outflow). All cohorts with birth, growing up and death streams are situated inside positive or negative feedbacks loops, while migration streams are defined as the proportion of the size of the given cohort or an independent parameter, for example the absolute values

Record #	Time	Year	Month	Day	M_0_4	M_5_9	M_10_14	M_15_19
1	38718	2008	1	1	24829.00	27000.00	32474.00	40307.00
2	39083	2007	1	1	26209.23	26588.40	31343.04	38897.24
3	39448	2008	1	1	27879.91	26153.67	30301 23	37109.93
4	39814	2009	1	1	29228.08	25875 55	29258.94	35582 25
5	40179	2010	1	1	30616.83	25943 23	28375 10	34178 45
8	40544	2011	1	1	31798.83	26442.83	27808.44	32868.80
7	40909	2012	1	1	32876.84	27291.57	27023.88	31678.34
8	41275	2013	1	1	33804.80	28390.78	26672.63	30574.47
9	41640	2014	1	1	34145.97	29601.05	28552.72	29584.94
10	42005	2015	1	1	34456.81	30790.82	28734.88	28727.90
11	42370	2016	1	1	34568.74	31898.90	27214.89	28021.25
12	42738	2017	1	1	33919.78	32856.81	27949.28	27502.24
13	43101	2018	1	1	33169.34	33825.22	28885.35	27202.85
14	43466	2019	1	1	32409.78	34149.16	29878.34	27147.72
15	43831	2020	1	1	31722.49	34373.33	30905.51	27344.77
16	44197	2021	1	1	31145,87	34295,85	31873,39	27781,11
17	44582	2022	1	1	30577.00	33972.28	32715.98	28418.28
18	44927	2023	1	1	29994.10	33489.09	33376.59	29202.37
19	45292	2024	1	1	29383.32	32926,70	33814,58	30069,53
20	45858	2025	1	1	28749.97	32338.44	34014.04	30952.37
21	46023	2028	1	1	28408.20	31740.38	33988.02	31788.65
22	46388	2027	1	1	28072,10	31143,13	33771,70	32512,71
23	46753	2028	1	1	27750,70	30559,82	33411,53	33084,47
24	47119	2029	1	1	27430.52	30017.54	32952.83	33472.82

Fig. 2. An excerpt from the results of the continuous simulation model.

of individuals. At the end of demographic simulation we receive multicolumn table that contains the predictions of cohorts sizes in subsequent years (2010–2030), (see Fig. 2).

Our research is based on the situation in a Polish administrative region called the Wrocław Region (WR). The demographic forecasts are usually prepared by the government scientific institutions and take into account the various combinations of population parameters, such as fertility rate (similar to birth rate), mortality (death) rate, life expectancy, rate or number of migrations, which are described using the qualitative categories expressed by: very high, high, average, low, very low.

In our study, we chose Wrocław area population and one of the population forecast option (see Table 1) developed by Polish Government Population Council for 2014–2050 [19] called in our other publications "Scenario 3". This option was randomly selected for examination.

 Table 1. The description of demographic parameters. One of the official population forecast, selected for our study.

Variant	Fertility rate	Mortality rate	Life expectancy	Migrations
"No 3"	High	Medium	Medium	Medium

In the further part of the paper we present the elements of the model that considers all the assumptions described above. Our main goal was to develop, implement and validate the operation of the proposed IT solution. It is clear that such a computer tool depends strongly on the structure of input data and during the verification/validation process it is necessary to consider different sets of input data, also coming from our previous research. Such an approach ensures the effectiveness of the whole scientific process. This paper however focuses only on some specific IT solutions and does not aim at the discussion of the results for the complete set of the population forecast options.

#### 3.3 Arrival Model

The second model was built in accordance with the discrete-event approach in Extendsim, too. It contains 36 hierarchical blocks, i.e. the same number as the number of main cohorts. The hierarchical blocks allow us to quickly build large models because each of their identical structure. Hierarchical blocks can be controlled by different parameters and can also represent multiplied stream sources. All outputs of hierarchical blocks are connected to a single output stream (see Fig. 3). In every source cohort the specific object (i.e. service request) has been assigned an attribute value "cohort number" that enables us to recognize the source cohort of that object. Each object may be linked to the particular parameter of random distribution, such as the service time or the code of the disease.



Fig. 3. An excerpt from the structure of the discrete-event model – "hierarchical blocks" level.

The data describing cohorts are read from the tables, for example the parameters of random exponential distributions that define time between subsequent requests. The simulated size of the population in a given cohort is calculated based on the intensity of the requests (historical, monthly) and the size of the population (historical, yearly).

#### 3.4 Integration of the Models

One of the challenges to be overcome when trying to integrate two different simulation approaches in one master model is the issue of mutual communication between two sub-models. The monthly intensity of patients arrivals is associated not only with historical monthly data from 2010 but also with the sizes of population cohorts in the end of the simulated year. Therefore, theoretically, each year should be simulated twice. First, the number of arriving patients should be generated according to the parameters describing every cohort and second, the simulation should be repeated using the coefficient calculated previously.

The next challenge to be overcome when performing hybrid simulation is ensuring the compliance between the deterministic continuous simulation and the stochastic discrete approach. In case of deterministic simulation repetitions are unnecessary. Hence it seems that the best option is to prepare the demographic forecasts by the continuous model and store the results in the external table which is at the same time the "input" table for discrete model.

#### 3.5 Spreadsheet Interface

We have developed the spreadsheet interface (MS Excel) to enable the fast and accurate modification of the parameters necessary to define the seasonality of patients arrivals. The user first selects a range of months for which the modified seasonality will be applied. In the next step the cohorts for the modifications are selected. Some cohorts may be excluded from the modification. For example, one can select months from February to April and only men's cohorts (see Fig. 4). It is also possible to select only cohorts with the highest number of arrivals (see Fig. 5).

coefficient					M_0_4	M_5_9	M_10_14	M_15_19
x	2	start; end	2	5	1	1	0	1
40179	31	2010	1	1	0,016434	0,011109	0,010292	0,011789
40210	28	2010	2	1	0,049167	0,030479	0,012055	0,025861
40238	31	2010	3	1	0,060818	0,044756	0,017633	0,028886
40269	30	2010	4	1	0,051995	0,033046	0,01605	0,026032
40299	31	2010	5	1	0,054218	0,044275	0,019036	0,034823
40330	30	2010	6	1	0,026503	0,02318	0,018353	0,01804
40360	31	2010	7	1	0,027277	0,018689	0,012739	0,018982
40391	31	2010	8	1	0,020475	0,01171	0,012307	0,017041
40422	30	2010	9	1	0,022731	0,014879	0,018317	0,014072
40452	31	2010	10	1	0,024415	0,014959	0,019108	0,0147
40483	30	2010	11	1	0,022563	0,012192	0,015654	0,013929
40513	31	2010	12	1	0,026031	0,014277	0,013135	0,012474
40544	31	2011	13	1	0,026031	0,014277	0,013135	0,012474

**Fig. 4.** An excerpt from the MS Excel interface. The aim is to indicate the largest stream intensity values (the smallest mean in random an exponential distribution) then decrease its intensity (coefficient X = 2) in months from 2 to 5 ("start; end") for cohorts M\_0\_4, M\_5\_9 and M\_15\_19.

Due to the very large number of resulting output records, i.e. millions of records that contain information about the time arrival and cohort's number, we have also developed an analytical tool in MS Excel spreadsheet to easily observe patients arriving in particular months.

		Number of cases (arrivals)								
Year	Month	M_0_4	M_5_9	M_10_14	M_15_19	M_20_4	M_25_29			
2010	1	488	277	286	413	347	489			
2010	2	730	380	335	453	386	514			
2010	3	903	558	490	506	550	574			
2010	4	772	412	446	456	459	469			
2010	5	805	552	529	610	551	683			
2010	6	787	578	510	632	550	650			
2010	7	810	466	354	665	512	784			
2010	8	608	292	342	597	428	627			
2010	9	675	371	509	493	506	548			
2010	10	725	373	531	515	550	680			
2010	11	670	304	435	488	456	546			
2010	12	773	356	365	437	391	514			

**Fig. 5.** An excerpt from the historical number of arrivals (2010). The months with the highest frequencies are highlighted.

# 4 Simulation Experiment

#### 4.1 Basic Assumptions

Simulation experiments were conducted according to the demographic scenario described earlier (see Table 1). We decided to study the effects of changes in the values of the seasonality indicators on the intensity of patients arriving to healthcare facilities. It seems that the seasonality is caused by the variabilities in morbidity trends separately for different cohorts during the year. We will conduct the research on the impact of hypothetical changes in the seasonal morbidity trends on the intensity of simulated arrivals to the healthcare system.

#### 4.2 Results and Discussion

We propose coefficient C as the independent variable and the historical intensity from 2010 as the reference intensity. By multiplying the reference value by the value of parameter C we would like to increase the number of arrivals in a specified month for a specified cohort.

The formula for calculations is as follows (1):

Parameter of exponential random distribution (time between arrivals) [hours]

(1)

- = (1 / historic number of arrivals in a given month in a given cohort)
- \* (given cohort size in 2010 year/cohort size)
- \* (number of hours in this month in 2010 / number of hours in a given month in a given year)

In our experiment the coefficient C is selected as the independent variable. For each cohort we found a month with the highest number of historical (2010) arrivals (see Fig. 5): for both gender the month of July is described by the highest numbers of arriving patients. This intensity is particularly often observed for the cohorts of the age

groups from 25 to 50 years. Historical data reveal also that the highest number of women older than 60 years registers in healthcare facilities in March. Several experiments were performed in order to check the conformity of the model with the historical data.

The coefficient C is multiplied by the number of arrivals in each of the highlighted month. The intensity of arrivals throughout the period from 2010 to 2030 relative to historical intensity (year 2010) was multiplied by a constant value in the range of 0.1 to 2 with step 0.1. This means that, for example, the coefficient C = 2 in the March 2010 causes that in simulated March 2010 we have 1620 arrivals in a cohort M\_0\_4 instead of 903 (but on average 1806). Figure 6 shows the simulated arrivals with the increased intensity (C = 2).

		Number o	f cases (si	mulated)				
year	r month	M_0_4	M_5_9	M_10_14	M_15_19	M_20_24	M_25_29	
201	5 1	404	234	299	519	405	489	
201	0 2	593	321	353	563	509	514	
201	0 3	1620	483	538	626	660	574	
201	0 4	703	333	475	538	554	469	
201	0 5	680	477	598	770	1354	683	
201	0 6	678	974	511	771	656	650	
201	0 7	712	393	379	1676	564	784	
201	0 8	535	237	339	696	517	627	
201	0 9	579	319	541	565	651	548	
201	0 10	634	280	1107	633	637	680	
201	0 11	533	257	504	610	611	546	
201	0 12	700	337	391	550	486	514	

**Fig. 6.** An excerpt from the simulated number of arrivals (2015). The value of factor C was increased (C = 2). The months with the highest frequencies are highlighted. It should be noted that the values in the table are affected not only by seasonality but also by changes in the population size as a result of continuous system simulation.

The parameters of interarrival time distributions were calculated based on the historical number of arrivals in the year 2010, separately for each calendar month and each age-gender cohort. The sizes of cohorts are extracted from historical data or – beyond the range of historical data – from the demographic simulation model.

The relationship between the total number of simulated arrivals (2010–2030) and the changing value of coefficient C is demonstrated in Fig. 7. The interesting phenomenon was observed. As expected, the higher coefficient C leads to the higher number of arrivals (in a statistical sense), however a noticeable irregularity can be seen when the small value of coefficient C is applied, i.e. C = 0.1. Smaller values of C reduce to almost zero the significance of the previously leading month. In Fig. 8 the histogram of the simulated frequency distribution of interarrival times resulting from the changes in coefficient C, is presented. The observed interarrival times are consistent with historical data (almost 200,000 arrivals in one year), the histogram shape corresponds to the exponential distribution and the basic statistics (as variance, not shown in the paper) are very close to each other within the tested range of coefficient C values (the experiment assumes that the Poisson process parameters change only at the turn of the month).



Fig. 7. The number of arrivals in the function of coefficient C. The almost linear relationship.



Fig. 8. The histogram of the frequency distribution of the simulated intervals in the function of the coefficient C.

The output values generated by the model are consistent with the historical data. The growth of C coefficient increases the intensity of the simulated arrival stream and the overall number of simulated arrivals to healthcare facilities. It can also be observed that a histogram of the interarrival time distribution preserves its original exponential pattern, however the parameters follow step-wise changes according to monthly seasonality and the trend of population size generated by the continuous model. The slight differences observed in the simulated values are the result of the fact that changes were introduced only in one month.

### 5 Summary

Our contribution is summarized as follows. We have developed the hybrid simulation model composed of two sub-models which were elaborated using different simulation paradigms. Both models, i.e. the population model based on the continuous approach and the arrivals model built with discrete-event methodology, are created on one IT platform (Extendsim). The overall aim of the simulation was to forecast future demand for healthcare services, taking into account the probable demographic changes. We were faced with the challenge of overcoming a large number of data, resulting from the experiments, when planning and conducting the simulation. The MS Excel interface (in VBA language) was developed to overcome these difficulties.

We performed the series of experiments to check the consistency of results with the assumption that the seasonality of incidences overlaps on population trends. We managed to demonstrate the correct response of the model to the modifications made on the independent variable. The modified values of the parameter C influenced the intensity of arrivals however the seasonal character of the arrivals was maintained.

Acknowledgements. This project was financed by the grant *Simulation modelling of the demand for healthcare services* from the National Science Centre, Poland, and was awarded based on the decision 2015/17/B/HS4/00306.

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